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A Quadratic-Semidefinite Relaxation Approach for DL OFDMA Resource Allocation Using Adaptive Modulation

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Abstract

This paper proposes two binary quadratic formulations for minimizing power subject to bit rate and sub-carrier allocation constraints over wireless downlink (DL) Orthogonal Frequency Division Multiple Access (OFDMA). The first model represents a restricted case in which users are allowed to use only one modulation size in each sub-carrier while the second, a more flexible real case in which they can use any size. We propose two semidefinite programming relaxations (SDP) and compare with the linear programs (LP) obtained by applying Fortet linearization method to the quadratic models. Numerical results show a total average tightness gain of 42.78 % and 97.17 % in the first and second case, respectively. Moreover we get near optimal bounds, in average, of 1 % for the second model over realistic data.

Keywords: *Orthogonal Division Multiple Access, downlink allocation, adaptive modulation, semidefinite programming.*

1 Introduction

When several users are connected to a Base Station (BS) a large number of signals use the wireless channel, therefore greater complexity is generated by the negative phenomena of Multiple Access Interference (MAI) and Multi-path distortions. OFDMA is a suitable technology for combating these negative phenomena and it is currently the type of modulation used in wireless multi-user systems such as IEEE 802.11a/g WLAN, in networks of fixed access such as IEEE 802.16a and also for mobile WiMax deployments networks ensuring high quality of service (QoS) requirements [1]. OFDMA divides the channel into several orthogonal narrow band frequencies forming sub-carriers (sub-channels) giving access to several users simultaneously. In order to solve the resource allocation problem of DL sub-carrier and power over OFDMA systems, several schemes and algorithms have been proposed [2]. In [3] for example, the problem of minimizing total power consumption with constraints on bit error rate (BER) for users requiring different services is formulated while in [4], a sub-carrier allocation algorithm is proposed to increase the total user data rates subject to BER, total transmission power and proportional rate constraints. In this work we deal with the problem of minimizing power subject to assignment sub-carrier constraints using adaptive modulation, which means varying the number of bits to be sent in the different sub-carriers. We state two quadratic formulations for this problem. A restricted one in which users are allowed to use one modulation size in its allotted sub-carriers and a more real flexible formulation in which they can use any modulation size in each sub-carrier. We derive two Semidefinite programming (SDP) relaxations to compare with the equivalent linear programs obtained by linearizing the quadratic models using Fortet linearization method [5].

SDP is a subfield of convex optimization concerned with the optimization of a linear objective function subject to the intersection between an affine set and the conic space generated by positive semidefinite matrices. SDP arises as a generalization of linear programming by replacing the vector of variables with a symmetric matrix and replacing the nonnegativity constraints with a positive semidefinite constraint. As in linear programming, SDP has also several important properties: convexity, it has a rich duality theory (although not as strong as linear programming), and admits theoretically efficient solution procedures based on iterating interior point methods. A strong link between SDP and combinatorial optimization has been established in last decades. We mention for example the work of Grötschel, Lovász and Schrijver [15] who investigated in detail both LP and SDP relaxations to combinatorial optimization problems. Later, Lovász and Schrijver [16] showed that SDP problems could provide tighter relaxations for binary programming problems. Other important works are the contributions of Nesterov and Nemirovski [17, 18] and Alizadeh [19] who have shown that interior point methods, pioneered by Karmarkar [20] for LP could be extended to SDP. Another recent and important work is due to Goemans and Williamson [6] who showed that SDP could be highly effective when finding good approximations to the maxcut problem, hence we use SDP to get tighter bounds due to its proven efficiency in combinatorial optimization. Moreover, actual SDP solvers use interior point algorithms with polynomial time complexity exploiting matrix sparsity [7, 11]. The contribution of this paper is mainly focused in the quality of the lower bounds obtained from a combinatorial optimization point of view rather than from a practical real implementation. The paper is organized as follows: Section 2 provides the general system description of the OFDMA allocation problem. Section 3 states the two new quadratic formulations for this problem and the equivalent linear formulations. Section 4 presents and explains the proposed SDP relaxations. Section 5 presents numerical results for the proposed SDP relaxations and those obtained by the equivalent linear models. Finally Section 6 provides some conclusions of this work.

2 System Description

We consider a single cell OFDMA wireless system composed by a base station (BS) and several users. The BS consists of a set of N sub-carriers that have to be assigned to a set of K users using a modulation size of $c \in \{1, \ldots, M\}$ bits in each sub-carrier. The BS must perform this allocation process over time in order to exploit the so-called multi-user diversity and hence increasing capacity of the system, although under the assumption of slow time varying channels [21]. The multi-user diversity occurs since sub-carriers perceive a large variation in channel gain which is different for each user, then each sub-carrier can vary its own transmission rate depending on the quality of the channel. The better the quality of the sub-channel is, the higher the throughput of bits that can be sent and the worse the less rate can be achieved. Therefore, for each user $k \in \{1, \ldots, K\}$ and each sub-carrier $n \in \{1, \ldots, N\}$, we may have a function $f(c_{k,n}, BER_k)$ depending on the amount of bits to be transmitted by the channel pair (k, n) taking into account the BER_k performance for each user. We can use the following formula for user k using subcarrier n with c bits.

$$P_{k,n}^c = \frac{f(c, BER_k)}{|\alpha_{k,n}|^2} \tag{1}$$

where $\alpha_{k,n}$ represents the time varying channel gain which can be modeled, for example as [8]:

$$\alpha_{k,n} = \sum_{i=1}^{L} w_i \exp^{j(2\pi f_i + \Phi_i)} \tag{2}$$

with L, w_i, f_i and Φ_i being the total number of incident waves, the amplitude, the doppler frequency and the initial phase of the incident wave, respectively. The main idea is to distribute efficiently and dynamically sub-carriers of the BS using adaptive modulation while minimizing total power in the system, but also having in mind the R_k bits requirement that each user has. The BS is faced with this NP-hard problem and once the decision is taken, the bits of each user are modulated into an adaptive M-PSK or M-QAM symbol to be subsequently combined using the inverse fast fourier transform (IFFT) into an

OFDMA symbol which is assumed to be transmitted through a slowly time-varying frequency-selective Rayleigh channel over a bandwidth B.

3 The Quadratic Allocation Schemes

We formulate the above problem as a quadratic integer programming problem for the two cases depending on the adaptive modulation condition.

3.1 A Restrictive Modulation Case: QIP1

In order to state a restrictive quadratic OFDMA model under the condition that each user must use only one size of modulation in its allotted sub-carriers we define the binary variables $x_{k,n}$ for user k using sub-carrier n and the binary variables $y_{k,c}$ for user k using a modulation size of c bits. Using the above power formula (1), the quadratic model we propose for this restrictive case can be stated as:

$$\text{QIP1}: \quad \min_{x_{k,n}, y_{k,c}} \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} P_{k,n}^{c} x_{k,n} y_{k,c} \tag{3}$$

$$\text{st:} \quad \sum_{n=1}^{N} \left[x_{k,n} \sum_{c=1}^{M} c \cdot y_{k,c} \right] = R_k \quad \forall k \tag{4}$$

$$\sum_{k=1}^{K} \left[x_{k,n} \sum_{c=1}^{M} y_{k,c} \right] \leq 1 \quad \forall n \tag{5}$$

$$x_{k,n}, y_{k,c} \in \{0, 1\} \tag{6}$$

In QIP1, (3) is the objective function meaning that if a sub-carrier n is assigned to user k using a modulation size of c bits, then $P_{k,n}^{c}$ has to be minimized. Constraint (4) is the bit rate constraint which uses index c as an integer adaptive modulation parameter to reach the R_k bits needed by each user. Constraint (5) ensures that each sub-carrier must be used by only one user at a time. Notice from this last constraint that the $\sum_{c=1}^{M} y_{k,c}$ should always be equal to one, otherwise a user would not be receiving any bits from the BS. This observation together with the binary definition of variables $y_{k,c}$ force the restrictive modulation condition to be satisfied. Besides, we also ensure that $\sum_{k=1}^{K} x_{k,n}$ is between zero and one which is equivalent to say that each sub-carrier must be used by at most one user at a time.

3.2 A Flexible Modulation Case: QIP2

Another quadratic model can also be stated for a more realistic flexible case in which each user might use any bit size of modulation in its allotted sub-carriers. To do this, we redefine variables $y_{k,c}$ of QIP1 changing the index set k by the index set n to get the binary variables $y_{n,c}$ which means now that each sub-carrier may use a different modulation size of c bits. The quadratic model QIP2 we propose for this second flexible case can be stated as follows:

$$\text{QIP2}: \quad \min_{x_{k,n}, y_{n,c}} \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{c=1}^{M} P_{k,n}^{c} x_{k,n} y_{n,c} \tag{7}$$

$$\text{st:} \quad \sum_{n=1}^{N} \left[x_{k,n} \sum_{c=1}^{M} c \cdot y_{n,c} \right] = R_k \quad \forall k \tag{8}$$

$$0 \leq \sum_{k=1}^{K} x_{k,n} \leq 1 \quad \forall n \tag{9}$$

$$0 \leq \sum_{c=1}^{M} y_{n,c} \leq 1 \quad \forall n \tag{10}$$

$$x_{k,n}, y_{n,c} \in \{0,1\} \tag{11}$$

where constraint (8) represents the bit rate constraint to reach the R_k bits needed by each user, constraint (9) ensures that each sub-carrier must be used by only one user at a time while constraint (10) represents a linear modulation constraint ensuring that a sub-carrier must not use more than one modulation size.

3.3 Integer Linear Formulations for QIP1 and QIP2

In order to compare QIP1 and QIP2 with an SDP relaxation, we get the optimal solutions of QIP1 and QIP2 by transforming them into equivalent Integer Linear Programming models we call hereby by IP1 and IP2, respectively. We introduce linearization variables $\varphi_{k,n}^c = x_{k,n}y_{k,c}$ in QIP1 to get [5]:

$$\text{IP1}: \quad \min_{x_{k,n}, y_{k,c}, \varphi_{k,n}^c} \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=1}^{M} P_{k,n}^c \varphi_{k,n}^c \tag{12}$$

$$\text{st:} \quad \sum_{n=1}^{N}\sum_{c=1}^{M} c\varphi_{k,n}^c = R_k \quad \forall k \tag{13}$$

$$0 \leq \sum_{k=1}^{K}\sum_{c=1}^{M} \varphi_{k,n}^c \leq 1 \quad \forall n \tag{14}$$

$$x_{k,n} \geq \varphi_{k,n}^c \quad \forall k,n,c \tag{15}$$

$$y_{k,c} \geq \varphi_{k,n}^c \quad \forall k,n,c \tag{16}$$

$$\varphi_{k,n}^c \geq x_{k,n} + y_{k,c} - 1 \quad \forall k,n,c \tag{17}$$

$$x_{k,n}, y_{k,c}, \varphi_{k,n}^c \in \{0,1\} \tag{18}$$

and linearization variables $\varphi_{k,n}^c = x_{k,n}y_{n,c}$ in QIP2 to get:

$$\text{IP2}: \quad \min_{x_{k,n}, y_{n,c}, \varphi_{k,n}^c} \quad \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=1}^{M} P_{k,n}^c \varphi_{k,n}^c \tag{19}$$

$$\text{st:} \quad \sum_{n=1}^{N}\sum_{c=1}^{M} c\varphi_{k,n}^c = R_k \quad \forall k \tag{20}$$

$$0 \leq \sum_{k=1}^{K} x_{k,n} \leq 1 \quad \forall n \tag{21}$$

$$0 \leq \sum_{c=1}^{M} y_{n,c} \leq 1 \quad \forall n \tag{22}$$

$$x_{k,n} \geq \varphi_{k,n}^c \quad \forall k,n,c \tag{23}$$

$$y_{n,c} \geq \varphi_{k,n}^c \quad \forall k,n,c \tag{24}$$

$$\varphi_{k,n}^c \geq x_{k,n} + y_{n,c} - 1 \quad \forall k,n,c \tag{25}$$

$$x_{k,n}, y_{n,c}, \varphi_{k,n}^c \in \{0,1\} \tag{26}$$

In IP1 and IP2, constraints (15)-(17) and constraints (23)-(25) are the linearization Fortet constraints. With these linear IP models, now we derive two SDP relaxations for each quadratic model QIP1 and QIP2.

4 The Semidefinite Relaxations

In this section we derive two SDP relaxations, one for QIP1 and one for QIP2. To do that, we define the set $\mathcal{S}_n = \{Z \in \mathbb{M}_n, Z = Z^T\}$ as the set of n square symmetric matrices and $\mathcal{S}_n^+ = \{Z \in \mathcal{S}_n, a \in \mathbb{R}^n, a^T Z a \geq 0\}$ as the set of symmetric matrices satisfying the condition of positive semidefiniteness [9]. We also recall that a set $\mathcal{C}$ is an affine space if the line through any two distinct points in $\mathcal{C}$ lies in $\mathcal{C}$, i.e. if for any two points $p_1, p_2 \in \mathcal{C}$ and $\theta \in \mathbb{R}$, we have $\theta p_1 + (1-\theta)p_2 \in \mathcal{C}$ [10].

4.1 A Semidefinite Relaxation for QIP1

In order to derive an SDP relaxation for QIP1, we define the following vector z as:

$$z^T = \begin{pmatrix} x_{1,1} & \cdots & x_{1,N} & \cdots & x_{K,N} & y_{1,1} & \cdots & y_{1,M} & \cdots & y_{K,1} & \cdots & y_{K,M} \end{pmatrix}$$

Then, let matrix Z be a symmetric positive semidefinite matrix defined by:

$$Z = \begin{pmatrix} W & z \\ z^T & 1 \end{pmatrix} \succeq 0$$

where $W = zz^T$. The first SDP relaxation can be written as follows:

$$\begin{aligned} \text{SDP1}: \quad & \min_{Z} \quad Trace(PZ) && (27) \\ & \text{st:} \quad Trace(U_k Z) = R_k \quad \forall k && (28) \\ & \qquad Trace(V_n Z) \leq 1 \quad \forall n && (29) \\ & \qquad Trace(\Gamma_{i,j} Z) \geq 0 \quad \forall i < j && (30) \\ & \qquad diag(W) = z && (31) \\ & \qquad Z \succeq 0 && (32) \end{aligned}$$

where the $Trace$ operator represents the usual inner product for matrices; ie, for matrices P and Z, we have

$$Trace(PZ) = \sum_i \sum_j P_{i,j} Z_{i,j}$$

Matrices P, U_k, V_n are symmetric matrices with entries equal to half the coefficients taken from (3), (4) and (5), respectively. Constraint (31) is a relaxation constraint for the condition of $z_i^2 = z_i$ for all i. The symmetric matrix $\Gamma_{i,j}$ in constraint (30) is used to have positive values in matrix Z and finally constraint (32) imposes the condition of matrix Z to be positive semidefinite. In SDP1, the objective function and constraints (28)-(31) are affine spaces and constraint (32) represents the conic space of positive semidefinite matrices.

4.2 A Tighter Semidefinite Relaxation for QIP2

In order to derive an SDP relaxation for QIP2, we simply redefine vector z as:

$$z^T = \begin{pmatrix} x_{1,1} & \cdots & x_{1,N} & \cdots & x_{K,N} & y_{1,1} & \cdots & y_{1,M} & \cdots & y_{N,1} & \cdots & y_{N,M} \end{pmatrix}$$

We can construct a similar matrix Z as in SDP1, but now according to the new vector z. Similarly we can reconstruct new symmetric matrices P, U_k, V_n, G_n and $\Gamma_{i,j}$ for all $i < j$ to state:

$$\begin{aligned}
\text{SDP2}: \quad & \min_{Z} && Trace(PZ) && (33)\\
& \text{st:} && Trace(U_k Z) = R_k, \quad \forall k && (34)\\
& && Trace(V_n Z) \leq 1, \quad \forall n && (35)\\
& && Trace(G_n Z) \leq 1, \quad \forall n && (36)\\
& && Trace(\Gamma_{i,j} Z) \geq 0 \quad \forall i < j && (37)\\
& && diag(W) = z && (38)\\
& && Z \succeq 0 && (39)
\end{aligned}$$

Unfortunately, the results for this new SDP2 formulation are not better than LP2, however it is possible to find a tighter SDP relaxation. Let's define for each n constraint in QIP2, the coefficient vectors ex_n, ey_n with each element representing the coefficient of each variable in vector z, then using the fact that $diag(W) = z$ we can write these constraints as $\text{Tr}(Diag(ex_n)W) \leq 1$ and $\text{Tr}(Diag(ey_n)W) \leq 1$ where we use the following proposition in order to strength SDP2.

Proposition 1. *For each n, $Trace([ex_n][ex_n]^T W) \leq 1$ is tighter than $Trace(Diag(ex_n)W) \leq 1$.*
Proof: *(See lemma 2.1 in [9]). To see this, we just focus in one variable, say $x_{k,n}$.*

We write: $\sum_{k=1}^{K} x_{k,n} \leq 1$ as $[ex_n]^T z \leq 1$, then $([ex_n]^T z)(z^T [ex_n]) \leq 1^2$ and $Trace([ex_n][ex_n]^T W) \leq 1$ ($W = zz^T$) since $[ex_n]^T z \geq -1$. Now, for any matrix $Q = W - zz^T \succeq 0$, we have $Trace([ex_n][ex_n]^T [Q + zz^T]) \leq 1$ wich is equivalent to $[ex_n]^T Q[ex_n] + ([ex_n]^T z)^2 \leq 1$ where the proof follows since $[ex_n]^T Q[ex_n] \geq 0$ due to the positive semidefiniteness of Q. ■

In order to use proposition 1, we add one zero row-column vector to the rank 1 positive semidefinite matrices $[ex_n][ex_n]^T$ and $[ey_n][ey_n]^T$ to state a tighter SDP relaxation we call hereby TSDP2:

$$\begin{aligned}
\text{TSDP2}: \quad & \min_{Z} && Trace(PZ) && (40)\\
& \text{st:} && Trace(U_k Z) = R_k, \quad \forall k && (41)\\
& && Trace([ex_n][ex_n]^T Z) \leq 1, \quad \forall n && (42)\\
& && Trace([ey_n][ey_n]^T Z) \leq 1, \quad \forall n && (43)\\
& && Trace(\Gamma_{i,j} Z) \geq 0 \quad \forall i < j && (44)\\
& && diag(W) = z && (45)\\
& && Z \succeq 0 && (46)
\end{aligned}$$

The only difference of TSDP2 compare to SDP2 are constraints (42)-(43).

5 Simulation Results

We solve IP1, LP1 and SDP1 first and then IP2, LP2 and TSDP2. Here LP1, LP2 are the relaxations of IP1 and IP2 respectively, and SDP1, TSDP2 are the proposed semidefinite relaxations for QIP1 and QIP2. For the numerical experiments we simulate one random power sample varying the number of users for different fixed number of sub-carriers. This is a realistic assumption in OFDMA systems since the bandwidth of a single channel can span from 1.25MHz to 20MHz and is closely linked to the number of sub-carriers to be used in the Discrete Fast Fourier Transform (DFFT) which can take values of 32, 64, 128, 512, 1024 [2]. Besides, the relation between the number of users and sub-carriers usually satisfy $K << N$ [13]. The maximum number of bits to be transmitted in each sub-carrier is set to $M = 4$. These are also very common modulation sizes when using M-PSK or M-QAM modulations in OFDMA systems [12]. We use only one power sample due to the high computational effort when computing integer solutions, however some results for small and medium size instances averaged over 50 power

Table 1: Instances for the Restricted Modulation Case

Size			LP1 Relaxation		SDP1 Relaxation	
#	n	k	# Constraints	# Variables	# Constraints	# Variables
1	32	4	2884	656	693	10585
2	32	6	4310	984	1023	23653
3	32	8	5736	1312	1353	41905
4	32	10	7162	1640	1683	65341
5	32	12	8588	1968	2013	93961
6	32	14	10014	2296	2343	127765
7	64	4	5732	1296	1365	37401
8	64	6	8566	1944	2015	83845
9	64	8	11400	2592	2665	148785
10	64	10	14234	3240	3315	232221
11	64	12	17068	3888	3965	334153
12	64	14	19902	4536	4615	454581
13	128	4	11428	2576	2709	140185
14	128	6	17078	3864	3999	314821
15	128	8	22728	5152	5289	559153
16	128	10	28378	6440	6579	873181
17	128	12	34028	7728	7869	1256905
18	128	14	39678	9016	9159	1710325

Table 2: Instances for the Flexible Modulation Case

Size			LP2 Relaxation		TSDP2 Relaxation	
#	n	k	# Constraints	# Variables	# Constraints	# Variables
1	32	4	3140	768	837	33153
2	32	6	4550	1088	1159	51681
3	32	8	5960	1408	1481	74305
4	32	10	7370	1728	1803	101025
5	32	12	8780	2048	2125	131841
6	32	14	10190	2368	2447	166753
7	64	4	6276	1536	1669	131841
8	64	6	9094	2176	2311	205761
9	64	8	11912	2816	2953	296065
10	64	10	14730	3456	3595	402753
11	64	12	17548	4096	4237	525825
12	64	14	20366	4736	4879	665281
13	128	4	12548	3072	3333	525825
14	128	6	18182	4352	4615	821121
15	128	8	23816	5632	5897	1181953
16	128	10	29450	6912	7179	1608321
17	128	12	35084	8192	8461	2100225
18	128	14	40718	9472	9743	2657665

Figure 1: Greedy Heuristic for the Restricted Modulation Case

```
Input: x_Sdp1, x_Lp1, R_k     Output: Feasible Solutions for QIP1
for each sub-carrier
  Pick a uniform Random number r ∈ [0,1)
  if (x_Sdp1, x_Lp1 ≥ r) then
    x_Sdp1, x_Lp1 ← 1
  else
    x_Sdp1, x_Lp1 ← 0
  end if
end for
if(Not Feasible x_Sdp1, x_Lp1 due to (4),(5) or Not reached N_k)
  Randomly add or erase subcarriers
end if
for each user k of y_Sdp1, y_Lp1
  Determine the modulation size as: R_k/N_k
    y_Sdp1, y_Lp1 ← 1
end for
```

channel samples are also provided. The sizes of the instances we simulate and the number of constraints as well as the number of variables for LP1, SDP1, LP2 and TSDP2 relaxations are shown in tables 1 and 2 respectively. The number of variables in SDP1 is calculated according to the size of matrix Z, so if the order of this matrix is F, then the number of variables is $F(F+1)/2$ [7]. From a theoretically point of view, this is the correct number of variables in an SDP formulation, although only the non-zero entries are used and solvers usually exploit sparsity structure of the matrix. As an observation from these tables we can say that LP1 and LP2 have more constraints than SDP1 and TSDP2, but we have the opposite for the number of variables. On the other hand the number of variables and constraints for LP2 and TSDP2 are larger than for LP1 and SDP1. We simulate up to these number of SDP constraints since current solvers can not solve larger instances [11].

The numerical experiments for these instances are first run using random data for powers which we generate as

$$P_{k,n}^{c} = \frac{c \cdot Rand(k,n)}{M} \quad c \in \{1,\dots,M\} \tag{47}$$

and second; we simulate using more realistic data by means of equation (48) to compute the power matrix in (1) since it is the required transmission power for c bits/sub-carrier at a given BER with unity channel gain [14].

$$f(c_{k,n}, BER_k) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{BER_k}{4}\right)\right]^2 (2^c - 1) \tag{48}$$

where $Q^{-1}(x)$ is the inverse function of

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \tag{49}$$

The main difference of random and realistic data is that it is spread linearly and exponentially respectively. We use channel model (2) and without loss of generality, we set parameter values according to [13] as follows. The total number of incident waves is set to $L = 200$, doppler frequency is assumed to be $f_i = 30Hz$, for all i and the BER_k is set to 10^{-3} for each user. Finally, we set the power spectral density to $N_0 = 1/N$ dBW in each sub-carrier. The amplitude vector $(w_1, \dots, w_L)$ is assumed to be identically and Normally distributed in each component like $w_i \sim N(\mu = 0, \sigma^2 = 1)$. The initial phase Φ_i of the i^{th} incident wave is calculated as $\Phi_i = 2\pi\lambda(i)/\max_{\{i\}}\{\lambda(i)\}$, for all i where λ is a vector also assumed to be identically and Normally distributed $N(0,1)$. The assumption of these power matrices is also realistic since higher values are common when using higher modulations. A Matlab program is developed using

Table 3: Results for the Restricted Modulation Case over Random Data

Instance	LP1				SDP1			Gaps			
	Opt	Feas	Lb	Time	Feas	Lb	Time	OP$_{SDP1}$	OP$_{LP1}$	GH$_{SDP1}$	GH$_{LP1}$
1	1.66	6.21	0.97	1.30	3.33	1.27	7.9	0.23	0.42	1.63	5.38
2	0.75	6.74	0.43	0.33	4.83	0.45	22.1	0.40	0.42	9.70	14.55
3	0.82	7.03	0.46	0.38	4.01	0.58	52.7	0.29	0.45	5.87	14.41
4	0.64	5.58	0.36	0.42	5.50	0.37	94.8	0.42	0.43	13.85	14.31
5	0.22	2.44	0.20	0.45	2.34	0.21	149.7	0.04	0.09	10.14	11.2
6	0.20	3.25	0.08	0.53	3.05	0.09	306.4	0.55	0.60	32.88	39.62
Average	0.71	5.20	0.41	0.56	3.84	0.49	105.60	0.32	0.40	12.34	16.57
7	0.40	4.79	0.28	0.38	2.84	0.40	43.3	0	0.30	5.93	15.80
8	1.33	7.77	0.92	0.47	6.13	1.06	119.7	0.20	0.30	4.81	7.41
9	1.78	11.62	1.08	0.95	8.64	1.27	299.7	0.29	0.39	5.82	9.80
10	1.12	11.53	0.44	0.75	5.33	0.55	735.2	0.50	0.60	8.69	25.20
11	0.83	7.22	0.31	0.80	6.50	0.36	1443.4	0.56	0.62	17.05	22.29
12	0.48	5.94	0.38	0.77	4.53	0.41	1440.6	0.14	0.20	10.04	14.63
Average	0.99	8.14	0.56	0.68	5.66	0.67	680.31	0.28	0.40	8.72	15.85
13	0.71	7.58	0.52	0.59	2.25	0.56	162.8	0.21	0.26	3.05	13.54
14	2.77	11.67	2.25	0.83	4.19	2.39	711.8	0.14	0.19	0.75	4.18
15	1.27	9.14	0.89	0.86	3.37	1.17	1757.9	0.08	0.30	1.88	9.25
16	1.14	12.53	0.85	1.06	7.50	0.87	4356.4	0.24	0.26	7.62	13.81
17	0.78	6.75	0.58	1.23	6.45	0.61	8330.1	0.21	0.25	9.57	10.63
18	0.75	14.29	0.46	2.33	4.55	0.62	14325	0.17	0.38	6.33	30.06
Average	1.23	10.32	0.92	1.15	4.71	1.03	4940.70	0.17	0.27	4.86	13.57
Tot. Average	0.98	7.89	0.63	0.80	4.74	0.73	1908.90	0.25	0.35	8.64	15.33

Cplex 9.1 and Csdp [11] software for solving the IP's, LP's and SDP's models. We use a Pentium IV, 1.9GHz with 2 GBytes of RAM under windows. Result tables for the restricted and flexible modulation case show the optimum solutions for IP, feasible integer solutions obtained with a simple greedy heuristic, lower bounds for LP as for SDP and the cpu time in seconds for both relaxations. The gaps are calculated as

$$\mathrm{OP}_{SDP} = \left[\frac{Opt-SDP}{Opt}\right], \quad \mathrm{OP}_{LP} = \left[\frac{Opt-LP}{Opt}\right], \quad \mathrm{GH}_{SDP} = \left[\frac{Feas-SDP}{SDP}\right], \quad \mathrm{GH}_{LP} = \left[\frac{Feas-LP}{LP}\right].$$

5.1 Results for the Restricted Modulation Case: IP1, LP1 and SDP1

We generate feasible integer solutions, for QIP1 from LP1 and SDP1 using a simple greedy heuristic just to confirm the tightness of SDP over LP. The greedy heuristic is only intended to find a feasible solution in a fair manner rather than to find the optimal solution of QIP1. The algorithm for this heuristic is shown in Figure 1. It simply takes as input the R_k bits needed by each user, the relaxed sub-carrier allocation matrices x_{Sdp1}, x_{Lp1} from SDP1 and LP1, then it does a one randomized rounding iteration on its elements and corrects if there is no feasible solution. To generate these feasible solutions we put the values of $R_k = N_k T$ where $1 \leq T \leq M$. N_k is a random number of sub-carriers for each user in $1 \leq N_k \leq \lfloor N/K \rfloor$. Notice from this algorithm that the modulation used by a particular user can be easily determined as R_k/N_k since each user uses a unique modulation size.

In the case of random data, from table 3 we see that the average gaps for OP$_{SDP1}$ and OP$_{LP1}$ are 25% and 35 %, respectively. This means that we have a tightness gain of 28.57 % of SDP1 over LP1. This is somehow confirmed with the gain of 43.6 % obtained by using the greedy heuristic of Figure 1. We also see this gain from table 5 in which each instance has been averaged over 50 sample power matrices. Here OP$_{SDP1}$ and OP$_{LP1}$ are equal to 27% and 36%. This gives a tightness gain of 25 % of SDP1 over LP1. However, even when SDP1 is tighter than LP1, we are still far from the optimal solution. On the opposite, we have nearer optimal solutions when using realistic data. The average results of OP$_{SDP1}$ and OP$_{LP1}$ in table 4 are 6% and 14 % which gives a tightness gain of 57% of SDP1 over LP1. The greedy

Table 4: Results for the Restricted Modulation Case over Realistic Data

Instance	LP1				SDP1			Gaps			
	Opt	Feas	Lb	Time	Feas	Lb	Time	OP_{SDP1}	OP_{LP1}	GH_{SDP1}	GH_{LP1}
1	0.08	1.04	0.06	0.31	0.08	0.08	6.6	0	0.25	0	16.33
2	0.33	1.38	0.29	0.33	0.61	0.32	18.6	0.03	0.12	0.90	3.75
3	0.34	8.97	0.28	0.42	8.10	0.29	47.0	0.15	0.18	26.68	30.61
4	0.30	7.38	0.26	0.45	7.58	0.27	105.7	0.10	0.13	27.07	27.38
5	0.15	1.78	0.13	0.45	1.72	0.14	166.7	0.06	0.13	11.28	12.69
6	0.25	11.06	0.20	0.58	2.47	0.22	260.7	0.12	0.20	10.22	54.3
Average	0.24	5.26	0.20	0.42	3.42	0.22	100.88	0.07	0.16	12.69	24.17
7	0.88	9.87	0.68	0.41	5.59	0.79	39.9	0.11	0.23	6.09	13.47
8	0.30	10.95	0.27	0.50	5.28	0.29	128.0	0.04	0.10	17.14	39.30
9	0.37	5.92	0.35	0.61	5.54	0.36	349.2	0.02	0.05	14.38	15.91
10	0.69	6.68	0.59	0.80	4.94	0.63	658.0	0.08	0.14	6.84	10.32
11	0.31	9.93	0.28	0.78	4.16	0.29	1706.3	0.09	0.10	13.49	34.27
12	0.21	11.43	0.19	0.84	9.79	0.20	2421.5	0.08	0.10	48.91	58.92
Average	0.46	9.13	0.39	0.65	5.88	0.42	883.81	0.07	0.12	17.80	28.69
13	0.56	11.93	0.46	0.61	6.43	0.52	260.7	0.07	0.17	11.36	24.93
14	0.92	21.96	0.79	0.94	3.70	0.90	651.7	0.02	0.14	3.11	26.79
15	1.03	18.61	0.88	1.19	13.08	0.97	1695.7	0.06	0.15	12.49	20.26
16	0.83	19.88	0.71	1.63	17.98	0.79	3917.4	0.05	0.14	21.83	26.84
17	0.47	42.68	0.40	1.34	23.70	0.43	9105.1	0.08	0.14	54.11	105.70
18	0.55	18.54	0.48	2.52	11.00	0.51	17647	0.07	0.12	20.56	37.62
Average	0.72	22.26	0.62	1.37	12.64	0.68	5546.3	0.05	0.14	20.57	40.35
Tot. Average	0.47	12.22	0.40	0.81	7.31	0.44	2177	0.06	0.14	17.02	31.07

Table 5: Average Results for the Restricted Modulation Case over Random Data

Instance	LP1				SDP1			Gaps			
	Opt	Feas	Lb	Time	Feas	Lb	Time	OP_{SDP1}	OP_{LP1}	GH_{SDP1}	GH_{LP1}
1	1.03	4.56	0.65	0.39	4.30	0.78	6.21	0.25	0.36	6.63	7.87
2	0.75	4.28	0.48	0.44	3.87	0.56	18.54	0.23	0.33	7.56	9.62
3	0.67	4.96	0.44	0.40	4.44	0.50	39.62	0.25	0.34	9.80	12.48
4	0.56	4.47	0.34	0.46	4.18	0.38	76.23	0.32	0.39	10.82	13.21
5	0.39	3.99	0.22	0.46	3.65	0.24	135.75	0.37	0.42	15.32	17.95
6	0.46	4.90	0.27	0.48	4.47	0.30	220.49	0.35	0.40	15.88	18.43
7	1.78	8.26	1.24	0.42	6.83	1.47	41.01	0.18	0.31	5.28	7.06
8	1.17	7.62	0.76	0.49	6.56	0.90	125.08	0.24	0.35	9.33	11.74
9	0.99	8.17	0.64	0.62	7.14	0.76	301.15	0.24	0.35	10.27	13.36
10	0.84	8.37	0.52	0.65	7.31	0.62	585.17	0.27	0.37	12.35	16.76
Tot. Average	0.86	5.96	0.56	0.48	5.28	0.65	154.93	0.27	0.36	10.32	12.85

Table 6: Average Results for the Restricted Modulation Case over Realistic Data

Instance	LP1				SDP1			Gaps			
	Opt	Feas	Lb	time	Feas	Lb	time	OP_{SDP1}	OP_{LP1}	GH_{SDP1}	GH_{LP1}
1	0.34	6.34	0.27	0.32	4.42	0.29	5.71	0.10	0.17	14.10	25.51
2	0.27	13.27	0.22	0.36	5.34	0.23	17.12	0.12	0.17	23.60	47.70
3	0.27	14.48	0.23	0.42	6.92	0.24	39.07	0.09	0.12	26.68	66.17
4	0.23	6.29	0.20	0.46	6.56	0.21	74.87	0.08	0.13	30.23	30.45
5	0.18	4.37	0.16	0.49	4.26	0.17	136.42	0.05	0.11	24.05	26.31
6	0.24	5.02	0.21	0.59	4.78	0.22	222.18	0.08	0.12	20.72	22.90
7	0.74	21.64	0.47	0.44	9.62	0.53	38.81	0.11	0.19	19.46	37.14
8	0.51	18.80	0.43	0.57	15.65	0.47	112.21	0.07	0.15	32.88	43.15
9	0.46	15.66	0.39	0.64	11.23	0.42	265.94	0.08	0.14	26.03	40.95
10	0.37	28.85	0.33	0.71	20.45	0.34	536.49	0.07	0.11	57.86	84.81
Tot. Average	0.36	13.47	0.29	0.50	8.92	0.31	144.88	0.08	0.14	27.56	42.50

Figure 2: Greedy Heuristic for the Flexible Modulation Case

```
Input: x_{Tsdp2}, x_{Lp2}, y_{Tsdp2}, y_{Lp2}, R_k    Output: Feasible Solutions for QIP2
for each sub-carrier
   Pick a uniform Random number r ∈ [0,1)
   if (x_{Tsdp2}, x_{Lp2}, y_{Tsdp2}, y_{Lp2} ≥ r) then
      x_{Tsdp2}, x_{Lp2} ← 1
      y_{Tsdp2}, y_{Lp2} ← 1
   else
      x_{Tsdp2}, x_{Lp2} ← 0
      y_{Tsdp2}, y_{Lp2} ← 0
   end if
end for
if(Not Feasible solutions due to (8),(9),(10))
   Randomly { add or erase sub-carriers
            { adjust y_{Tsdp2}, y_{Lp2} for each user to reach R_k
end if
```

heuristic again confirms it with a gain of 45.22 %. In order to have a more global idea of the average about the improvement achieved by SDP1 over LP1, from table 6 we see that the tightness gain is 42.85% and 35.15 % when using the greedy heuristic.

On the other hand, even when CSDP solver uses interior point algorithms with polynomial time complexity [11], cpu times are still larger for SDP1 than for LP1, although they are no to so big for small and medium size instances.

5.2 Results for the Flexible Modulation Case: IP2, LP2 and TSDP2

For the second flexible modulation case, we generate feasible integer solutions for QIP2 from LP2 and TSDP2 relaxations using a similar greedy heuristic as in the above case. Again, this heuristic is intended only to find a feasible solution in a fair manner rather than to find the optimal solution of QIP2. The algorithm takes as input the R_k bits needed by each user, the relaxed sub-carrier allocation matrices x_{Tsdp2}, x_{Lp2} and the relaxed modulation matrices y_{Tsdp2}, y_{Lp2} from TSDP2 and LP2, then it does a one randomized rounding iteration on their elements and corrects not only by randomly adding or erasing sub-carriers but also adjusting the values of matrices y_{Tsdp2}, y_{Lp2} until a solution is feasible. The main difference of the greedy algorithm shown in Figure 2 compared to the above shown in Figure 1 is in matrices y_{Tsdp2} and y_{Lp2} which now are more flexible because they change in each sub-carrier ($y_{n,c}$) instead of changing in each user ($y_{k,c}$).

For random data, results of table 7 show that the average gaps for OP$_{TSDP}$ and OP$_{LP2}$ are 5e-3% and 17 %, which is a high tightness gain of 98.23 % for TSDP2 over LP2. A gain of 98.05 % is achieved using now the greedy heuristic of Figure 2. Similarly, table 9 gives a tighteness gain of 99.95% for TSDP2 over LP2, hence we say that solutions are almost optimal when using random data. Fortunately, we have also excelent results when using realistic data. The average results of OP$_{SDP1}$ and OP$_{LP1}$ in table 8 are 1% and 37 % respectively, so the tightness gain achieved is 97.29 % for TSDP2 over LP2. The greedy heuristic gives a gain of 94.05 %. The average improvement from table 10 gives a tightness of 97.14% and 83.88 % using the greedy heuristic. Finally we have the same situation for the cpu times. Here we recall that the main purpose of this paper is to find lower bounds rather than to solve the problem for a real application. Certainly, future research should be devoted in finding faster algorithms with lower complexity than interior point methods such as those proposed, for example, in [22] and in the references therein.

Table 7: Results for Flexible Modulation Case over Random Data

Instance	LP2				TSDP2			Gaps			
	Opt	Feas	Lb	time	Feas	Lb	time	OP_{TSDP2}	OP_{LP2}	GH_{TSDP2}	GH_{LP2}
1	2.37	3.72	1.90	0.31	2.39	2.29	12.05	0.03	0.20	0.04	0.95
2	1.97	3.56	1.63	0.36	1.97	1.97	30	0	0.17	0	1.18
3	2.64	5.06	2.18	0.41	2.64	2.64	75.81	0	0.17	0	1.32
4	1.92	3.84	1.56	0.41	2.29	1.91	121.55	1e-3	0.19	0.20	1.46
5	1.88	5.45	1.36	0.45	1.90	1.80	151.94	0.04	0.28	0.05	3.00
6	2.61	5.36	2.17	0.50	2.61	2.61	258.14	0	0.17	0	1.46
Average	2.23	4.49	1.80	0.40	2.30	2.20	108.24	0.01	0.19	0.04	1.56
7	8.58	11.70	7.39	0.39	8.58	8.58	99.11	0	0.14	0	0.58
8	5.60	8.94	4.60	0.48	5.60	5.60	261.88	0	0.18	0	0.94
9	5.33	11.50	4.36	0.67	5.38	5.32	608.27	1e-3	0.18	9e-3	1.64
10	4.31	10.75	3.42	0.66	4.31	4.31	1109.3	0	0.21	0	2.14
11	3.41	9.35	2.53	0.75	3.59	3.35	1228.7	0.01	0.26	0.07	2.70
12	2.63	5.11	2.32	3.72	2.72	2.61	1816.7	6e-3	0.12	0.04	1.20
Average	4.97	9.55	4.10	1.11	5.03	4.96	853.99	2e-3	0.18	0.01	1.53
13	8.70	12.91	8.05	0.59	8.70	8.70	593.97	0	0.08	0	0.60
14	8.28	15.94	7.02	0.80	8.51	8.23	1685.3	5e-3	0.15	0.03	1.27
15	5.90	13.77	4.80	0.94	5.97	5.89	3259	1e-3	0.19	0.01	1.87
16	5.40	15.10	4.64	2.44	5.77	5.39	7579.5	1e-3	0.14	0.07	2.25
17	9.38	19.16	7.67	1.38	9.38	9.38	7298.8	0	0.18	0	1.50
18	5.67	13.93	4.94	1.47	6.15	5.66	8842.7	1e-3	0.13	0.08	1.82
Average	7.22	15.13	6.18	1.27	7.41	7.20	4876.5	1e-3	0.14	0.03	1.55
Tot. Average	4.81	9.73	4.03	0.92	4.91	4.79	1946.3	5e-3	0.17	0.03	1.54

Table 8: Results for Flexible Modulation Case over Realistic Data

Instance	LP2				TSDP2			Gaps			
	Opt	Feas	Lb	time	Feas	Lb	time	OP_{TSDP2}	OP_{LP2}	GH_{TSDP2}	GH_{LP2}
1	0.61	0.98	0.38	0.36	0.61	0.61	19.70	0	0.37	0	1.56
2	0.39	0.70	0.26	0.36	0.55	0.38	41.47	0.01	0.34	0.46	1.72
3	0.68	1.48	0.32	0.45	0.70	0.66	93.53	0.02	0.53	0.06	19.38
4	0.43	0.87	0.24	0.52	0.53	0.42	167.14	0.02	0.45	0.27	2.66
5	0.28	0.45	0.16	0.52	0.37	0.26	173.38	0.04	0.42	0.42	1.82
6	0.22	0.44	0.14	0.55	0.33	0.21	258.38	0.04	0.38	0.55	2.22
Average	0.43	0.82	0.25	0.46	0.51	0.42	125.60	0.02	0.41	0.29	4.8933
7	0.79	1.63	0.61	0.44	0.79	0.79	112.67	0	0.22	0	1.66
8	1.26	1.92	0.68	5.77	1.40	1.25	415.36	0.01	0.47	0.12	1.84
9	0.57	1.19	0.41	0.66	0.57	0.57	633.56	0	0.29	0	1.95
10	0.51	1.00	0.33	0.77	0.51	0.51	1102.2	0	0.35	0	2.02
11	0.74	1.11	0.40	1.20	0.74	0.74	1544.2	0	0.46	0	1.76
12	0.56	0.95	0.34	1.14	0.78	0.54	1864.9	0.02	0.39	0.44	1.81
Average	0.73	1.30	0.46	1.66	0.79	0.73	945.48	5e-3	0.36	0.09	1.84
13	1.92	3.17	1.18	0.86	2.64	1.83	859.59	0.04	0.39	0.44	1.69
14	1.20	2.26	0.82	1.14	1.20	1.20	2051.9	0	0.32	0	1.76
15	0.78	1.82	0.60	1.64	0.78	0.78	3888.8	0	0.24	0	2.04
16	1.44	2.19	0.82	1.92	1.44	1.44	11928	0	0.43	0	1.68
17	0.75	1.70	0.53	1.83	0.75	0.75	7193.9	0	0.30	0	2.20
18	0.98	1.70	0.62	2.45	1.41	0.97	14363	4e-3	0.36	0.45	1.73
Average	1.17	2.14	0.76	1.64	1.37	1.16	6714.2	7e-3	0.34	0.14	1.85
Tot. Average	0.78	1.42	0.49	1.25	0.89	0.77	2595.1	0.01	0.37	0.17	2.86

Table 9: Average Results for Flexible Modulation Case over Random Data

Instance	LP2				TSDP2			Gaps			
	Opt	Feas	Lb	time	Feas	Lb	time	OP_{TSDP2}	OP_{LP2}	GH_{TSDP2}	GH_{LP2}
1	3.30	4.80	2.84	0.44	3.32	3.28	13.96	5e-3	0.13	0.01	0.70
2	2.54	4.28	2.15	0.35	2.62	2.51	30.97	0.01	0.15	0.04	1.02
3	2.57	4.79	2.08	0.41	2.65	2.55	70.86	0.01	0.19	0.04	1.34
4	2.24	4.69	1.80	0.41	2.42	2.21	128.82	0.01	0.20	0.10	1.66
5	2.02	4.32	1.65	0.73	2.20	1.98	164.90	0.02	0.18	0.12	1.70
6	2.11	5.79	1.69	0.61	2.44	2.06	237.17	0.02	0.20	0.21	2.53
7	8.85	12.39	7.69	0.39	8.85	8.85	99.77	0	0.12	0	0.62
8	5.74	10.20	4.66	0.49	5.96	5.72	264.48	2e-3	0.18	0.04	1.19
9	4.57	9.16	3.88	1.51	4.67	4.56	556.13	3e-3	0.15	0.02	1.41
10	3.92	8.76	3.16	0.70	4.04	3.90	1049.4	4e-3	0.19	0.03	1.79
Tot. Average	3.78	6.91	3.16	0.60	3.91	3.76	261.64	8e-3	0.16	0.06	1.39

Table 10: Average Results for Flexible Modulation Case over Realistic Data

Instance	LP2				TSDP2			Gaps			
	Opt	Feas	Lb	time	Feas	Lb	time	OP_{TSDP2}	OP_{LP2}	GH_{TSDP2}	GH_{LP2}
1	0.41	0.75	0.28	0.38	0.59	0.40	18.02	4e-3	0.29	0.56	1.79
2	0.35	0.66	0.23	0.36	0.35	0.35	39.55	0	0.33	0	1.89
3	0.34	0.60	0.21	0.49	0.47	0.33	85.71	0.01	0.38	0.45	1.87
4	0.30	0.56	0.18	0.48	0.41	0.29	153.53	0.02	0.37	0.45	2.00
5	0.29	0.49	0.17	0.74	0.40	0.27	189.65	0.05	0.40	0.47	1.95
6	0.28	0.56	0.16	0.73	0.39	0.26	274.43	0.04	0.41	0.50	2.51
7	0.80	1.56	0.55	0.59	0.80	0.80	116.05	0	0.28	0	1.88
8	0.71	2.04	0.46	0.76	0.71	0.71	316.89	0	0.34	0	3.21
9	0.55	1.04	0.35	0.72	0.83	0.54	645.56	4e-3	0.34	0.59	1.98
10	0.57	1.10	0.35	0.91	0.81	0.56	1233	7e-3	0.36	0.47	2.11
Tot. Average	0.46	0.93	0.29	0.61	0.57	0.45	307.23	0.01	0.35	0.34	2.11

6 Conclusions

In this paper, we proposed two binary quadratic and two SDP formulations for minimizing power subject to bit rate and sub-carrier allocation constraints over wireless DL OFDMA using adaptive modulation. Two linear relaxations were derived applying Fortet linearization method to the quadratic models. Numerical results showed a total average tightness gain of 42.78 % and 97.17 % of SDP over LP. We also achieved near optimal bounds, in average of 1 % for the second quadratic model when using SDP over realistic data. The Best results are achieved for this last case which is better since it highly approaches the conditions of real systems. Future research should be devoted to find lower complexity algorithms such as those proposed in [22].

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